Now if we have regard to the order of λ, μ, ν, ξ, there are $(p-1)^3$ ways of satisfying the original congruence. For each of three parameters can be assigned in $p-1$ ways, and the congruence determines the remaining one. Hence we have

$$(p-1)^3=n_4+{}_3C_4n_3+6n_{2_1}+2{}_2C_4n_{2_2}+{}_4P_4n_1,$$
$$(p-1)^3=n_4+4n_3+6n_{2_1}+12n_{2_2}+24n_1.$$

Substituting the values found above for n_{2_2}, n_{2_1}, n_3, n_4, we get, when $p\equiv1 \pmod 4$,

$$n_1=\tfrac{1}{24}(p^3-9p^2+29p-21)$$

and when $p\equiv3 \pmod 4$,

$$n_1=\tfrac{1}{24}(p^3-9p^2+29p-33).$$

Hence, when $p>5$ is a prime of the form* $4l+1$ the number (N) of sets of solutions of $\lambda+\mu+\nu+\xi\equiv0 \pmod{p-1}$ is,

$$N=\tfrac{1}{24}(p^3-9p^2+29p-21)+\tfrac{1}{2}(p-3)^2+p-3+p-5+4$$
$$=\tfrac{1}{24}(p^3+3p^2+5p-9).$$

But when p is of the form $4l+3$,

$$N=\tfrac{1}{24}(p^2-9p^2+29p-33)+\tfrac{1}{2}(p-3)^2+p-2+p-3+2$$
$$=\tfrac{1}{24}(p^3+3p^2+5p+3).$$

GEOMETRY.

284. Proposed by JOHN JAMES QUINN, Ph. D., Warren, Pa.

a) Suppose that two radii R and r, whose center is the origin, revolve with uniform angular velocities 3θ and θ, respectively. What is the equation of the locus of P, the projection parallel to the X axis of the extremity of the radius r on the radius R produced if necessary.

b) Apply this curve to the trisection of an angle.

c) Suppose the ratio of their velocities is $n\theta:\theta$. Show how we can effect the multisection of an angle.

Solution by A. H. HOLMES, Brunswick, Maine.

a) Take O, the center of the circle, radius a, as the origin of coördinates. Then taking any angle θ, we shall have $r\sin3\theta=a\sin\theta$.

$\therefore r=\dfrac{a\sin\theta}{\sin3\theta}$ is the equation of the locus of the point P.

b) Construct the curve $r=\dfrac{a\sin\theta}{\sin3\theta}$. On the circumference of the circle

*The values $p=3$, 5 constitute exceptions in the method employed. The final result does not hold for $p=5$. But, by inspection, when $p=5$, $N=10$; and when $p=3$, $N=3$.

take an arc equal to any angle ψ from axis of x. Draw a line from the upper limit of the arc to the origin. From the point P, where this line cuts the curve $r=\dfrac{a\sin\theta}{\sin 3\theta}$, draw a line parallel to axis of x cutting the given arc. From this point draw a line to O making an angle$=\frac{1}{3}\psi$.

c) The equation of the locus of P would be $r=\dfrac{a\sin\theta}{\sin n\theta}$, and the multisection would be similar to the trisection.

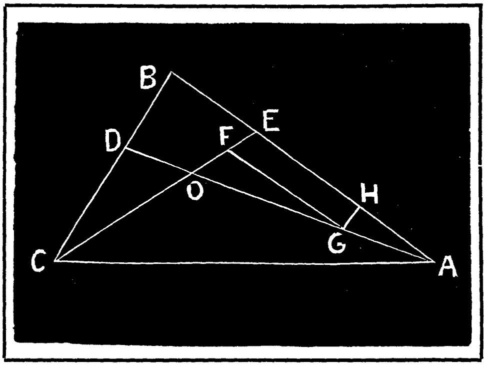
NOTE. By projecting parallel to the y-axis Dr. Zerr obtains in a similar way the locus $r=\dfrac{a\cos\theta}{\cos n\theta}$, and effects the n-section of the angle in a manner similar to the above. ED.

285. Proposed by G. E. BROCKWAY, Nashua, N. H.

Prove without the aid of the circle, that if the bisectors of the angles of a triangle be drawn, the greatest bisector falls on the least side.

I. Solution by ALFRED H. PARROTT, North Dakota Agricultural College, N. D.

Given scalene triangle ABC, and bisectors of angles A and C, supposing $\angle C>\angle A$. If $\angle C>\angle A$, then side $AB>$side BC, and we are to prove bisector AD on side $BC>$bisector CE on side AB. If $\angle C>\angle A$, $\angle OCA>\angle OAC$ [If wholes are unequal, then halves, etc.].



$\therefore AO>OC$ [If angles of a triangle, etc.]. Now if $OD>$ or $=OE$, the proposition is evident. But suppose $OD<OE$. Then on OE take $OF=OD$, and on OA take $OG=OC$, and draw FG. Then draw GH parallel to CE.

$\triangle FOG=\triangle COD$ [Three sides on one equal respectively, etc.]. Then $\angle FGO=\angle DCO>\angle EAO$ [Halves of unequals, etc.].

Consider $\triangle FGO$ and $\triangle EAO$; $\angle O\equiv\angle O$, $\angle FGO>\angle EAO$ [Previous proof]. Therefore $\angle GFO<\angle AEO$.

A line drawn parallel to FG and through E will then intercept HG between H and G and HG is therefore $>EF$.

Now $\angle AHG>\angle HAG$, for $\angle AHG=\angle AEC$, [sides respectively parallel], $\angle AEC=\angle B+\angle ECB$ [Exterior angle of a triangle equals sum of, etc.]. But $\angle HAG<\angle ECB$, and therefore $<\angle B+\angle ECB$, and hence $<\angle AHG$.

$\therefore AG>HG>EF$, or $AO-GO>OE-OF$,

$AO+OF>OE+GO$, $AO+OD>EO+OC$.

Hence $AD>EC$. Q. E. D.